When Can Communication Lead to Efficiency?*

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Abstract

We study games with incomplete information and characterize when a feasible outcome is Pareto efficient. We show that any outcome with excessive randomization over actions is inefficient. Generically, efficiency requires that the total number of actions taken across states be strictly less than the sum of the number of players and states. We then examine the efficiency of equilibrium outcomes in communication models. Generically, a cheap talk outcome is efficient only if it is pure. When the sender's payoff is state-independent, it is efficient if and only if the sender's most preferred action is chosen with certainty. In natural buyer–seller settings, Bayesian persuasion outcomes are inefficient across a wide range of priors and preferences. Finally, we show that our results apply to mechanism design problems with many players.

1 Introduction

An outcome is Pareto efficient if there is no other feasible outcome that makes at least one player strictly better off without making any other player worse off. Efficiency is a natural and desirable property in strategic situations. However, when players have conflicting interests, efficiency is far from guaranteed. One natural mechanism for overcoming inefficiency is communication. *Can strategic communication lead to Pareto efficiency?*

Communication plays a central role in many economic settings, yet it remains unclear when it leads to efficient outcomes. Existing models of strategic communication, such as

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cheap talk and Bayesian persuasion, mainly characterize equilibrium outcomes rather than examine whether they are Pareto efficient. We identify simple conditions for efficiency based on the number of actions taken across states and show that excessive randomization leads to inefficiency.

We examine games with incomplete information, where the state of the world is drawn from a common prior and players' payoffs depend on both the state and the chosen actions. We then focus on two-player sender–receiver games, where the sender observes the state and sends a message, while the receiver chooses an action after observing it. More information helps the receiver make better choices, but the sender may strategically conceal or distort information to influence the outcome. This tension lies at the heart of whether communication can lead to efficiency.

In particular, we focus on two models of strategic information transmission: cheap talk (Crawford and Sobel, 1982) and Bayesian persuasion (Kamenica and Gentzkow, 2011). In both, the sender communicates strategically to influence the receiver's action. The key distinction between the two models is that in Bayesian persuasion, the sender can commit to his signaling policy, while in cheap talk, he cannot. Our aim is to determine the efficiency of the equilibrium outcomes in both models.

1.1 Overview of Results

First, we establish a connection between ex-ante and ex-post efficiency using convex geometry. An outcome specifies a probability distribution over actions for each state. We show that an outcome is ex-ante efficient if and only if the state-contingent outcomes are ex-post efficient for all states and compatible (Proposition 1). An outcome is compatible if it maximizes a common positive linear functional over the feasible payoff sets of all states. The key argument is that the feasible set under the prior equals the Minkowski sum of the feasible sets across all states, and that maximizers of a linear functional are preserved under Minkowski addition.

Using this connection, we provide a necessary and sufficient condition for an outcome to be ex-ante efficient (Proposition 2). The condition depends solely on the players' payoff functions and can be verified by examining state-wise deviations from the recommended actions. An outcome is efficient if and only if there is no convex combination of feasible deviations, taken across all states, that makes every player weakly better off and at least one player strictly better off. In other words, no Pareto improvement across states is possible.

In Theorem 1, we provide a necessary condition for efficiency based on the number of actions taken across states. Generically, an outcome is efficient only if the total number of actions played across all states is strictly less than the sum of the number of players and the number of states. Any excessive randomization across actions necessarily leads to inefficiency, as it either violates compatibility or ex-post efficiency.

For intuition, consider the two-player sender-receiver model. We classify outcomes into three categories: pure, quasi-pure, and mixed. An outcome is pure if a deterministic action is taken in every state. It is quasi-pure if a deterministic action is taken in all but one state, where a binary action is taken. And, an outcome is mixed if it is neither pure nor quasi-pure. The bound for two players tells us that an outcome is efficient only if it is pure or quasi-pure. A mixed outcome fails to be efficient because it is either incompatible or ex-post inefficient in some state. First, consider the case where two or more actions are taken in multiple states. In any such state, the state-contingent outcome can be ex-post efficient only if it maximizes a unique linear functional determined by the actions taken. However, compatibility requires that the state-contingent mixed outcomes across all these states maximize the same linear functional. Each state determines its own unique functional, and any generic perturbation of the payoff matrix violates this requirement. Second, if more than two actions occur in a state, the outcome lies on the Pareto frontier of that state's feasible payoff set only if the corresponding payoff vectors are collinear. Again, this condition is not robust to perturbations, and therefore the outcome is ex-post inefficient. Using this result, we provide conditions under which the equilibrium outcomes in cheap talk and Bayesian persuasion are inefficient.

First, we consider the cheap talk model, where the sender cannot commit to a signaling policy. This lack of commitment imposes strict incentive constraints on the sender. We show that, generically, an equilibrium is efficient only if it is pure (Proposition 3). Any stochastic outcome requires the sender to be indifferent between multiple actions in a given state. However, generically, the receiver will strictly prefer one of these actions, leading to inefficiency. Next, when the sender's payoff is state-independent, we show that an equilibrium is efficient if and only if the sender's most preferred action is chosen with certainty (Proposition 4). In this case, any non-trivial equilibrium induces a posterior belief with positive weight on a state where both the sender and receiver prefer the sender's preferred action over the equilibrium outcome. Crucially, this action is taken with certainty only in a babbling equilibrium, implying that any equilibrium where communication affects the receiver's action is inefficient.

Second, we study Bayesian persuasion in a natural class of buyer–seller interactions. The seller provides information to a buyer who decides whether, and if so which, product to purchase. The buyer wants to choose the best available product, if any is worth buying, while the seller cares only about selling products with a high profit. To maximize his payoff, the seller tries to persuade the buyer to purchase a product even when none is valuable to the buyer. However, because his recommendations must satisfy the buyer's obedience condition, the seller may need to randomize his recommendations when no product is worth buying.

Using this intuition, we show that for a wide range of priors and preferences (Proposition 5 and 6), the Bayesian persuasion outcome is necessarily mixed and, hence, inefficient.

Finally, our insights also apply beyond two-player games. In particular, we visit the ranking-based peer selection mechanisms introduced in Niemeyer and Preusser (2024). In this environment, extremal mechanisms are inherently stochastic, making randomization unavoidable. Under mild assumptions, our bound on the number of actions is violated, implying that the resulting outcome is generically inefficient (Proposition 7).

1.2 Related Literature

Our work contributes to the literature on communication between an informed sender and an uninformed receiver. In cheap talk (Crawford and Sobel, 1982), the sender's message is unverifiable, while in Bayesian persuasion (Kamenica and Gentzkow, 2011), the sender commits to how the message is generated.[1] Ichihashi (2019) analyzes how restrictions on the sender's information in Bayesian persuasion affect the players' welfare. His analysis, which is limited to binary actions, shows that the Bayesian persuasion outcome is always efficient. However, our findings reveal that efficiency cannot be guaranteed when there are more than two actions. Relatedly, Rayo and Segal (2010) analyze the problem of finding disclosure rules that maximize a weighted sum of sender profits and receiver surplus. Cheng et al. (2024) show that for the sender to benefit from ambiguity in persuasion, at least two induced outcomes must be Pareto ranked, that is, both players agree on their payoff ranking. Doval and Smolin (2024) analyze a welfare function over a heterogeneous population and characterize the set of feasible welfare outcomes achievable through information policies. They characterize the Pareto frontier of this set using Bayesian persuasion problems.

Given its importance, several papers have focused on identifying the conditions that lead to Pareto efficient outcomes. These include works that explore the conditions under which Nash equilibria can be efficient or inefficient, such as Case (1974), Dubey (1986) and Cohen (1998). Another line of research looks at efficiency from a learning perspective, developing adaptive procedures that lead to efficient outcomes, such as Arieli and Babichenko (2012), Pradelski and Young (2012), Marden et al. (2014), and Jindani (2022). Arieli et al. (2017) study commitment procedures that result in efficiency in extensive form games. In a closely related paper, Rudov et al. (2025) analyze when a Nash equilibrium can be improved by a correlated equilibrium, exploiting the convex polytope structure of correlated equilibria. Like us, they derive geometric conditions that restrict the extent of randomization. However, they focus on improvability (or efficiency) within the set of equilibria, whereas we study efficiency

[1] There has been some work on selecting equilibria in cheap talk games that are Pareto dominant (see Crawford and Sobel, 1982, Antić and Persico, 2023). Our focus differs as we examine efficiency relative to all feasible payoffs, not just the set of equilibrium payoffs.

relative to the set of *all* outcomes. Closer to our setting with incomplete information, they show that a Bayesian Nash equilibrium is generically an extreme point of the set of Bayes correlated equilibria if and only if it is pure. In our setting, however, even a pure Bayesian Nash equilibrium may fail to be efficient.

2 Model and Preliminaries

We consider a game with incomplete information with $k \geq 2$ players. The state of the world is drawn from a finite set Ω according to a common prior $p \in \text{int}(\Delta\Omega)$. Each player $i \in \{1, \dots, k\}$ has a finite set of actions A_i, and we write $A = \prod_{i=1}^{k} A_i$ for the set of pure action profiles. Each player i has private information represented by a type $t_i \in T_i$, where T_i is finite. Let $T = T_1 \times \cdots \times T_k$ denote the set of type profiles with distribution $\pi : \Omega \to \Delta T$. Each player has a payoff function $u_i : \Omega \times A \to \mathbb{R}$, and we assume the collection of payoffs $(u_1, \dots, u_k)$ is *generic*, meaning that the properties we establish hold for all bounded payoffs except on a subset of Lebesgue measure zero.

An *outcome* is a mapping $\mu : \Omega \to \Delta A$, which assigns to each state $\omega \in \Omega$ a probability distribution $\mu(\cdot \mid \omega)$ over action profiles. Equivalently, one can think of a mediator who observes the realized state and recommends an action profile (possibly at random) to the players, which they follow. Crucially, the recommendation need not be incentive compatible; we evaluate efficiency relative to the set of all feasible outcomes.

The *payoff vector* induced by outcome μ under prior p is

$$u(\mu) \;=\; \sum_{\omega \in \Omega} p(\omega) \sum_{a \in A} \mu(a \mid \omega) \, \big(u_1(\omega, a), \dots, u_k(\omega, a)\big). \tag{1}$$

The set of feasible payoff vectors given prior p is defined as

$$F_p = \{\overline{u} \in \mathbb{R}^k : \overline{u} = u(\mu), \text{ for some outcome } \mu : \Omega \to \Delta A \,\}. \tag{2}$$

This set is a convex polytope. Its extreme points correspond to a mediator providing deterministic recommendations: in each state, the mediator recommends a pure action to every player.

Definition 1. *Given a compact convex set of feasible payoffs $F \subseteq \mathbb{R}^k$, a vector $\overline{u} \in F$ is efficient if there does not exist another feasible payoff vector $\overline{v} \in F$ such that $\overline{v} \geq \overline{u}$, with a strict inequality for at least one component.*

In our setting, a feasible payoff vector is efficient if and only if it maximizes a positive weighted sum of the players' payoffs.[2] For a convex polytope $F \subseteq \mathbb{R}^k$ and for any vector $n \in \mathbb{R}^k$, denote by

$$S(F; n) := \{x \in F \mid n^T x = \max_{y \in F} n^T y\} \tag{3}$$

the set of maximizers x of the inner product $n^T x$ over F.

In our setup, we can identify the set of feasible payoffs with the set of outcomes. Thus, we can analyze efficiency in terms of outcomes instead of payoff vectors.

Definition 2. *An outcome $\mu : \Omega \to \Delta A$ is **efficient** if $u(\mu)$ is efficient with respect to F_p.*

Given an outcome μ, we refer to $\mu(\omega) \in \Delta A$ and $u(\mu \mid \omega) \in \mathbb{R}^k$ as the outcome and payoff vector in state ω, respectively. Let

$$F_\omega = \mathrm{Co}\big((u_1(\omega, a), \ldots, u_k(\omega, a)) : a \in A\big). \tag{4}$$

denote the feasible payoff vectors in state ω.[3] Any outcome can be decomposed in terms of its state-contingent outcomes. This follows as the set of the feasible payoff vectors given a prior can be written as a unique Minkowski sum of the set of the feasible payoff vectors for each state:[4]

$$F_p = \sum_{\omega \in \Omega} p(\omega) F_\omega. \tag{5}$$

Our notion of efficiency is defined before uncertainty is resolved, that is, before the state is realized (ex-ante). However, efficiency can also be evaluated after uncertainty is resolved, once the state is realized (ex-post). An outcome μ is ex-post efficient in state ω if its induced outcome $\mu(\omega)$ is efficient with respect to the set of feasible payoff vectors in that state.

Definition 3. *An outcome $\mu : \Omega \to \Delta A$ is **ex-post efficient** in state ω if $u(\mu \mid \omega)$ is efficient with respect to F_ω.*

Efficiency implies ex-post efficiency but not the other way around. For the two notions to coincide, the state-contingent outcomes must satisfy an additional condition, which we call *compatibility*.

Definition 4. *An outcome $\mu : \Omega \to \Delta A$ is **compatible** if there exists a strictly positive $n \in \mathbb{R}^k_{++}$, such that $u(\mu \mid \omega) \in S(F_\omega; n)$ for all $\omega \in \Omega$.*

[2]This equivalence fails for general set of feasible payoff vectors but can be approximated by "near" weighted sum of the players' payoffs, as shown in Che et al. (2024). In our case, the equivalence holds because the set of feasible payoffs is a convex polytope.

[3]where $\mathrm{Co}(A)$ stands for convex hull of set A.

[4]The Minkowski sum of two sets A and B is given by $A + B = \{a + b \mid a \in A, b \in B\}$.

Compatibility ensures the existence of a common positive vector n that is maximized across all state-contingent feasible payoffs F_ω by the outcome μ.

Proposition 1. *An outcome $\mu : \Omega \to \Delta A$ is efficient if and only if it is ex-post efficient in all states and compatible.*

Proof. The characterization relies on the following relation for the Minkowski sum of convex compact sets (see Fukuda, 2004):

$$S(F_1 + ... + F_k; n) = S(F_1; n) + ... + S(F_k; n) \text{ for any } n \in \mathbb{R}^k \tag{6}$$

where, F_i is a compact convex set for all $i = 1, ..., k$ and $F_1 + F_2$ denotes the Minkowski sum of the sets F_1 and F_2.

Each point in the Pareto frontier of the set F_p maximizes some positive linear functional $n^T x$ over F_p. Given $F_p = \sum_\omega p(\omega)F_\omega$, and using Equation (6), the payoff vector $u(\mu)$ lies on the Pareto frontier of F_p if and only if, for all $\omega \in \Omega$, the payoff vector $u(\mu \mid \omega)$ lies on the Pareto frontier of F_ω and there exists a common positive vector n that is maximized. Ex-post efficiency ensures that the outcome maximizes a positive linear functional over the feasible payoff set for each state, while compatibility ensures that the same linear functional can be used across all states.

□

We illustrate the notions of ex-ante and ex-post efficiency using the following two player example.

Example 1. *Consider a game with two players: the sender and the receiver. Consider the state space $\Omega = \{\omega_0, \omega_1\}$ and the receiver's action space $A = \{a_0, a_1, a_2, a_3, a_4\}$. The sender's and receiver's payoffs are given by the following matrix:*

	a_0	a_1	a_2	a_3	a_4
ω_0	$(2, 9)$	$(10, 8)$	$(0, 6.4)$	$(3, 4)$	$(1, 0)$
ω_1	$(2, 0)$	$(10, 4)$	$(0, 6.4)$	$(3, 8)$	$(1, 9)$

We analyze the efficiency of three pairs of outcomes and priors $p = \mathbb{P}(\omega_1)$:

(a) $p = 0.10$*: an outcome where actions a_0 and a_1 are taken in ω_0 and action a_1 is taken in ω_1;*

(b) $p = 0.30$*: an outcome where action a_1 is taken with certainty in both states;*

(c) $p = 0.70$*: an outcome where action a_1 is taken in ω_0, and actions a_1 and a_4 are taken in ω_1.*

As we will later see, these outcomes correspond to the Bayesian persuasion outcomes for the respective priors.

The feasible payoff vectors for states ω_0, ω_1, and the prior p are represented by the red, blue and orange regions in Figure 1, respectively. We find that for (a) the outcome is ex-post efficient in both states but not compatible, for (b) the outcome is ex-post efficient in both states and compatible and for (c) the outcome is not ex-post efficient in state ω_1. The outcome in (a) is not compatible because the unique normal n_{ω_0} does not belong to the normal cone spanned by $\overline{n}_{\omega_1}$ and $\underline{n}_{\omega_1}$, i.e., $n_{\omega_0} \notin cone\{\overline{n}_{\omega_1}, \underline{n}_{\omega_1}\}$. The outcome is Pareto efficient in (b), but not in (a) and (c) (see the respective polytopes F_p in Figure 1). Thus, this exemplifies that an outcome is efficient if and only if it is ex-post efficient in all states and compatible.

Next, we provide a necessary and sufficient condition for an outcome to be efficient. This condition is determined solely by the payoff functions of the players. To check for efficiency, one must look at the possible change in pair of payoffs when deviating from the recommended action for all possible states. Define

$$d_\mu(\omega, a) := \big(u_1(\omega, a) - u_1(\omega, \mu(\omega)), \cdots, u_k(\omega, a) - u_k(\omega, \mu(\omega))\big) \tag{7}$$

as the deviation in state ω when action profile a is taken instead of the recommended profile $\mu(\omega) \in \Delta A$. Let

$$\mathcal{D}_\mu = \{\tilde{d} \in \mathbb{R}^k : \tilde{d} = d_\mu(\omega, a) \text{ for some } \omega \in \Omega, a \in A\} \tag{8}$$

denote the set of all deviations given outcome μ. And, let

$$\text{cone}(\mathcal{D}_\mu) := \Big\{ \sum_{d \in \mathcal{D}_\mu} \lambda_d \, d : \lambda_d \geq 0 \text{ for all } d \in \mathcal{D}_\mu \Big\}. \tag{9}$$

denote the cone generated by the set of deviations $\mathcal{D}_\mu$. An outcome is efficient if and only if no convex combination of deviations across states leads to a Pareto improvement.

Proposition 2. *An outcome $\mu : \Omega \to \Delta A$ is efficient if and only if*

$$cone(\mathcal{D}_\mu) \cap \mathbb{R}^k_+ = \{\mathbf{0}\}. \tag{10}$$

Proof. ($\Rightarrow$) We prove by contradiction. If μ is efficient, then there exists $n \in \mathbb{R}^k_{++}$ such that $n \cdot d \leq 0$ for all $d \in \mathcal{D}_\mu$. Suppose instead that (10) does not hold. Then there exist non-negative coefficients $(\lambda_d)_{d \in \mathcal{D}_\mu}$ with $d_\lambda = \sum_{d \in \mathcal{D}_\mu} \lambda_d d \in \mathbb{R}^k_+ \setminus \{\mathbf{0}\}$. In particular, as $n \in \mathbb{R}^k_{++}$, this implies that $n \cdot d_\lambda > 0$. But $n \cdot d_\lambda = \sum_{d \in \mathcal{D}_\mu} \lambda_d (n \cdot d)$, so there exists some $d \in \mathcal{D}_\mu$ with $\lambda_d > 0$ for which $n \cdot d > 0$, leading to a contradiction.

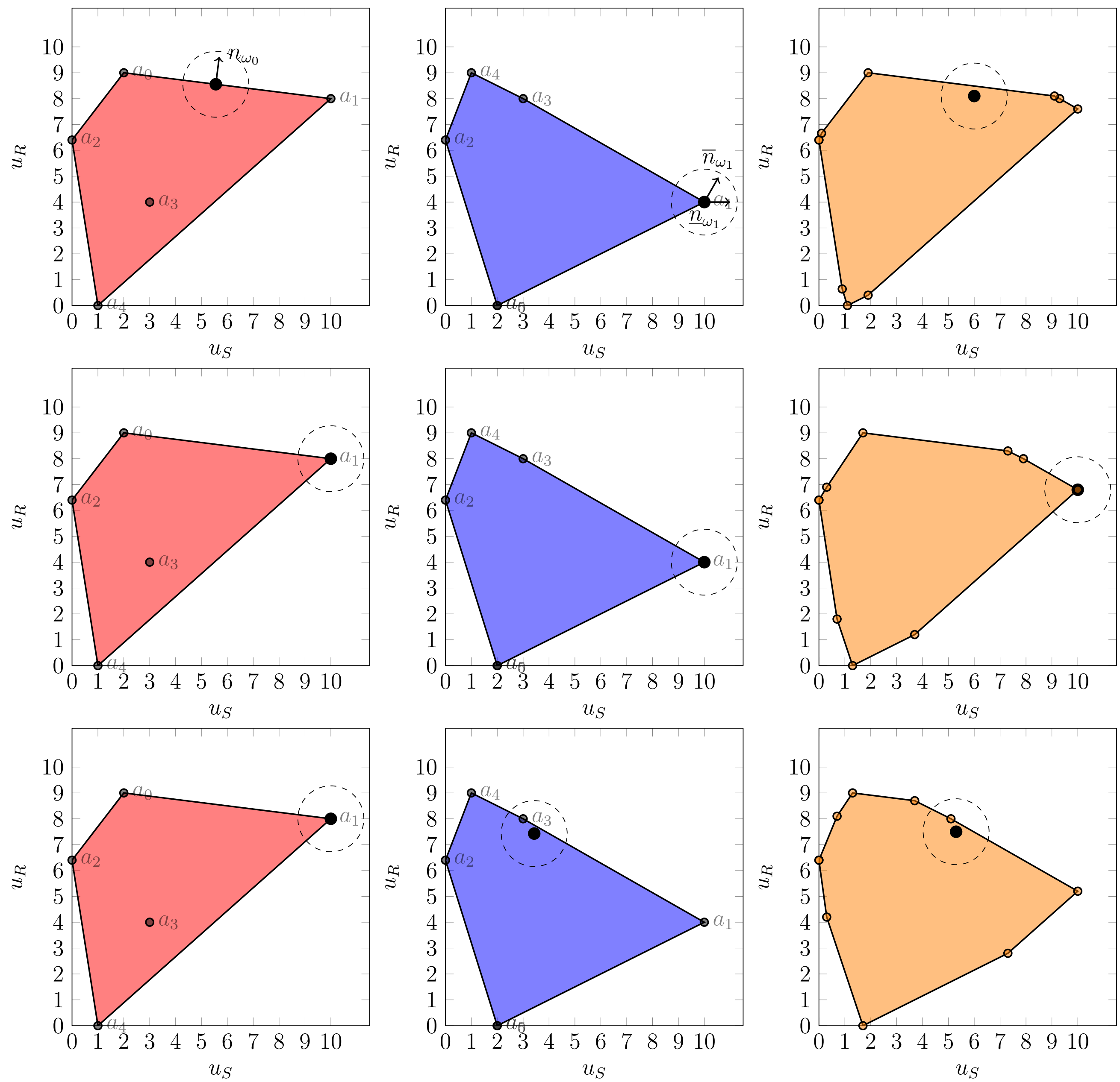

Figure 1: Outcomes (black node with circle) for respective priors: (a) $p = 0.10$, (b) $p = 0.30$ and (c) $p = 0.70$. The red region represents F_{ω_0}, the blue region represents F_{ω_1}, and the orange region represents F_p.

($\Leftarrow$) Assume (10) holds, i.e., no strictly positive vector lies in the cone generated by the deviations. Let A denote the $k \times |\mathcal{D}_\mu|$ matrix whose columns are the deviation vectors $d_\mu(\omega, a)$. By Mangasarian's Theorem (as stated in Perng, 2017), for any real matrix A exactly one of the following holds:

(i) There exists $x \geq \mathbf{0}$, $x \neq \mathbf{0}$ such that $Ax \geq \mathbf{0}$ (with at least one strictly positive component).

(ii) There exists $n > \mathbf{0}$ (strictly positive component wise) such that $n^\top A \leq \mathbf{0}$.

Since case (i) is ruled out by (10), case (ii) must hold. Thus, there exists $n \in \mathbb{R}^k_{++}$ such that $n \cdot d \leq 0$ for all $d \in \mathcal{D}_\mu$.

Fix any state ω and action $a \in A$. Then $n \cdot d_\mu(\omega, a) = n \cdot (u(\omega, a) - u(\mu \mid \omega)) \leq 0$. This implies $u(\mu \mid \omega) \in S(F_\omega; n)$. Hence μ is ex-post efficient in every state, and the same n supports all states, establishing compatibility. Thus, the outcome μ is efficient.

$\square$

Remark 1. *When $k = 2$, the condition reduces to: there is no deviation with $d_\mu(\omega, a) \geq \mathbf{0}$ (strict in at least one component), and for any pair of deviations $(d, \tilde{d})$ where d benefits player 1 and $\tilde{d}$ benefits player 2, the internal angle between d and $\tilde{d}$ is at most $180°$. Equivalently, $\tilde{d} \cdot d^\dagger \geq 0$, where $d^\dagger$ denotes the vector obtained by rotating d clockwise by $90°$. In particular, if $d = (x, -y)$ with $x, y \geq 0$, then $d^\dagger = (-y, -x)$.*

The efficiency condition depends solely on the support of the outcome and is independent of the weight of randomization and the prior. This has two implications. First, if an outcome is efficient (or inefficient) for a prior $p \in \text{int}(\Delta\Omega)$, it remains so for all interior priors. Second, the support of the Bayesian persuasion outcome (for each state) remains fixed within a convex region, with only the weight of randomization varying. Thus, if an equilibrium outcome is efficient (or inefficient) for a given prior, it holds across all priors in that convex region.

We describe outcomes by the number of actions taken in each state. This will play a fundamental role in our main result. Given an outcome μ and a state ω let $|\mu(\omega)|$ denote the size of the support of $\mu(\omega) \subset A$, namely the number of actions that are taken with positive probability given state ω. Our main result provides a necessary condition for efficiency. In words, efficiency requires that the overall number of actions taken across states be strictly less than the sum of the number of players and states; excessive randomization necessarily leads to inefficiency. Intuitively, efficiency requires that all state-contingent outcomes share a common strictly positive outer normal. When too many actions are played across all states, such a normal cannot exist.

Theorem 1. *Generically, an outcome $\mu : \Omega \to \Delta A$ is efficient only if*

$$\sum_{\omega \in \Omega} |\mu(\omega)| < k + |\Omega|. \tag{11}$$

Proof. Recall that the set of feasible payoffs in state ω and given prior p equals

$$F_\omega = \mathrm{Co}\Big(u(\omega, a) : a \in A\Big) \subset \mathbb{R}^k, \tag{12}$$

$$F_p = \mathrm{Co}\Big(u(\mu) : \mu : \Omega \to A\Big) \subset \mathbb{R}^k, \tag{13}$$

where $u(\omega, a) = \big(u_1(\omega, a), \ldots, u_k(\omega, a)\big)$ and $u(\mu) = \sum_{\omega,a} p(\omega)\mu(a \mid \omega)u(\omega, a)$.

Given any state ω, suppose the outcome mixes $|\mu(\omega)|$ pure action profiles $a_1, \ldots, a_{|\mu(\omega)|}$. An outcome μ is efficient if and only if it is ex-post efficient in every state and compatible. This follows from the fact that $F_p = \sum_\omega p(\omega)F_\omega$.

For the outcome μ to be ex-post efficient in state ω, the induced payoff vector $u(\mu \mid \omega)$ must maximize some strictly positive linear functional $n \in \mathbb{R}^k_{++}$ over F_ω. Equivalently, it must lie on a face of F_ω supported by some outer normal $n \in \mathbb{R}^k_{++}$. This requires the existence of some $n \in \mathbb{R}^k_{++}$ such that

$$n \cdot \big(u(\omega, a_i) - u(\omega, a_1)\big) = 0 \qquad \text{for } i = 2, \ldots, |\mu(\omega)|, \tag{14}$$

generically resulting in $|\mu(\omega)| - 1$ independent linear equations. Hence, the face has dimension $|\mu(\omega)| - 1$, and its outer normal cone has dimension $k - (|\mu(\omega)| - 1)$. Denote this outer cone by $N_\omega(\mu)$.

For the outcome μ to be compatible, there must exist a single nonzero positive normal that belongs to the respective normal cones of all states:

$$\bigcap_{\omega \in \Omega} N_\omega(\mu) \neq \emptyset. \tag{15}$$

Each state with $|\mu(\omega)|$ mixed actions adds $|\mu(\omega)| - 1$ independent constraints that the normal must satisfy. Generically, the dimension of the intersection of the outer normals is

$$\dim\Big(\bigcap_{\omega \in \Omega} N_\omega(\mu)\Big) = k - \sum_{\omega \in \Omega}(|\mu(\omega)| - 1) = k + |\Omega| - \sum_{\omega \in \Omega} |\mu(\omega)|. \tag{16}$$

If $\sum_\omega |\mu(\omega)| \geq k + |\Omega|$, the intersection is empty, implying that no common positive normal exists. Thus, the outcome is incompatible and inefficient.

□

Remark 2. *The aggregate bound implies a state-wise bound:* $|\mu(\omega)| \leq k$ *for all* $\omega \in \Omega$*. Generically, an outcome is ex-post efficient in a state only if the number of action profiles taken in that state is weakly less than the number of players.*

Remark 3. *For the two-player sender-receiver model, outcomes can be categorized as pure (a deterministic action is taken in every state), quasi-pure (a deterministic action is taken in every state except one, where two actions are taken with positive probability), or mixed. The bound* $\sum_\omega |\mu(\omega)| < 2 + |\Omega|$ *implies that, generically, an outcome is efficient only if it is pure or quasi-pure.*

Remark 4. *The same condition also characterizes when an outcome generically lies on any face of the polytope* F_p*, not only on the Pareto frontier. This is because the argument relies only on properties of the Minkowski sum of polytopes and their faces. Pareto efficiency only requires that the outer normal is strictly positive.*

The necessary condition in Theorem 1 is far from sufficient. For instance, in Example 1, the outcome is quasi-pure (satisfying the bound) in cases (a) and (c), yet it remains inefficient. Even if all state-contingent outcomes are pure and ex-post efficient, the overall outcome can still be inefficient. Efficiency is guaranteed when a player's most preferred outcome—one that is ex-post efficient in every state—is induced. In the example, this occurs when the signaling policy is fully revealing and the receiver takes his optimal action in each state, or when the sender's preferred action is chosen in every state, as in case (b) of Example 1.

Type-contingent decision rules: Following Bergemann and Morris (2019), outcomes can be viewed as the result of type-contingent decision rules $\sigma : T \times \Omega \to \Delta A$, where actions depend on both the type profile and the state. Since payoffs depend only on states and actions, the type profile and its distribution does not change the set of feasible payoff vectors. Type profiles become relevant only when incentive compatibility constraints are imposed.

A **Bayes correlated equilibrium (BCE)** consists of such type-contingent decision rules that satisfy obedience constraints. A **Bayesian Nash equilibrium (BNE)** is the subset of BCE in which each player randomizes over actions as a function of their type ($\sigma_i : T_i \to \Delta A_i$). If the distribution of type profiles $\pi : \Omega \to \Delta T$ has full support (assigns positive probability to every type at every state), then even pure decision rules may assign mixed actions across states.[5]

Corollary 1. *Assume* $\pi : \Omega \to \Delta T$ *has full support. If either*

[5] In a generic game, both the prior p and the type distribution π have full support.

(i) $\sigma : T \times \Omega \to A$ *and there are at least* k *states with types* $t, t' \in T$ *such that* $\sigma(t, \omega) \neq \sigma(t', \omega)$*, or*

(ii) $\sigma_i : T_i \to A_i$ *for all* $i = 1, \ldots, k$ *and at least* q *players use two or more actions across their types where* $|\Omega|(2^q - 1) \geq k$*,*

then the induced outcome μ *is generically inefficient.*

The corollary shows that BCE or BNE outcomes may even fail to be efficient when players follow pure decision rules. Differences in types impose distinct incentive constraints, which can generate excessive randomization and lead to inefficiency. Rudov et al. (2025) show that within the set of BCEs, a BNE is extreme if and only if it is pure.[6] In contrast, we show that efficiency may fail even for such pure rules when outcomes are evaluated relative to all feasible payoffs rather than only the set of equilibria payoffs.

3 Applications

3.1 Sender-Receiver Model

We now specialize the general framework to the two-player sender-receiver model. The environment consists of a finite state space Ω, a finite action space A, and a common prior $p \in \text{int}(\Delta\Omega)$. There are two players: the sender (S), who observes the realized state, and the receiver (R), who chooses an action after receiving a message. Payoffs are given by bounded functions $u_i : \Omega \times A \to \mathbb{R}$, for $i \in \{S, R\}$.

The receiver needs to take an action $a \in A$ where the unknown state $\omega \in \Omega$ is distributed according to prior $p \in \Delta\Omega$. The sender provides information about the state by sending a message according to a signaling policy $\sigma : \Omega \to \Delta M$, where M is a finite set of messages. We assume there are at least as many messages as actions or states, i.e., $|M| \geq \max\{|A|, |\Omega|\}$. The receiver on seeing message $m \in M$ drawn according to policy σ forms a posterior belief via Bayes' rule before taking an action. The receiver's strategy is denoted by $\tau : M \to \Delta A$. The sender-receiver setup includes the model of cheap talk and Bayesian persuasion, with the difference that in Bayesian persuasion the sender can commit to the signaling policy, while in cheap talk he cannot.

A strategy profile (σ, τ) induces an *outcome* $\mu : \Omega \to \Delta A$, specifying a probability distribution over actions for each state, where

$$\mu(a \mid \omega) = \sum_{m \in M} \sigma(m \mid \omega)\tau(a \mid m) \text{ for all } \omega \in \Omega, a \in A.$$

[6]For the case of complete-information games, they show that a Nash equilibrium is extreme in the set of correlated equilibria if and only if at most two players randomize.

A signaling policy is *direct* if the messages represent action recommendations ($M \equiv A$). Any feasible outcome can be achieved through some direct policy and an obedient receiver. To induce outcome μ, the sender recommends action a in state ω with probability $\mu(a \mid \omega)$, and the receiver follows the recommendation. Hence, as in our general framework, we can equivalently study outcomes $\mu : \Omega \to \Delta A$ without loss of generality.

The sender-receiver setup encompasses two canonical models of communication: *cheap talk* (Crawford and Sobel, 1982), where the sender cannot commit to a signaling policy, and *Bayesian persuasion* (Kamenica and Gentzkow, 2011), where the sender can commit ex ante.

3.1.1 Cheap talk

Cheap talk considers communication between an informed sender and uninformed receiver. The sender knows the state of the world and sends a message to persuade the receiver. In contrast to Bayesian persuasion, the model assumes that the sender cannot commit to how messages are generated. This imposes strict incentive constraints on the sender, ensuring that if he sends multiple messages with positive probability in a given state, he must be indifferent among them. The receiver's equilibrium condition is to choose the best response to the message given his posterior belief.

Definition 5 (Cheap talk outcome)**.** *A strategy profile (σ, τ) is a Perfect Bayesian Equilibrium (PBE) of cheap talk if it satisfies the following conditions:*

1. *(Sender) For every $\omega \in \Omega$ and for every $m \in supp(\sigma(\omega))$,*

$$u_S(\omega, \tau(m)) = \max_{m' \in M} u_S(\omega, \tau(m')). \tag{17}$$

2. *(Receiver) For each message $m \in M$, there exists a posterior belief $q_m \in \Delta\Omega$ such that, for every $a \in supp(\tau(m))$,*

$$a \in \arg\max_{a' \in A} \mathbb{E}_{q_m}[u_R(\omega, a')], \tag{18}$$

and for any $m \in \bigcup_{\omega \in \Omega} supp(\sigma(\omega))$, the posterior belief q_m is derived using Bayes' rule:

$$q_m(\omega) = \frac{p(\omega)\sigma(m \mid \omega)}{\sum_{\omega'} p(\omega')\sigma(m \mid \omega')}. \tag{19}$$

Unlike Bayesian persuasion, cheap talk exhibits a multiplicity of equilibria. In particular, a *babbling equilibrium* always exists, in which no communication takes place and the receiver plays his best response to the prior. Note that in the case of cheap talk, we cannot restrict attention to direct signaling policies, as some equilibria may require randomizing between actions for a given message.

First, we show that, generically, a cheap talk outcome can be efficient only if it is pure. This is a stronger result than the one stated in Theorem 1, as any stochastic outcome, including quasi-pure, is inefficient.

Proposition 3. *Generically, a cheap talk outcome $\mu : \Omega \to \Delta A$ is efficient only if it is pure.*

Proof. We prove by contradiction. Suppose there exists an efficient cheap talk outcome that is not pure.

First, consider the case where the signaling policy is necessarily stochastic. So, there exists $\omega^* \in \Omega$ and messages $m_1, m_2 \in M$ such that $\sigma(m_1 \mid \omega^*) \cdot \sigma(m_2 \mid \omega^*) > 0$. Note that $\tau(m_1) \neq \tau(m_2)$, since otherwise the same outcome could be induced using a single message. By Theorem 1, if more than two actions are played in a given state then the outcome is inefficient for a generic set of payoffs. Therefore, assume exactly two actions, a_1 and a_2, are played with positive probability. The sender's equilibrium condition implies that

$$u_S(\omega^*, a_1) = u_S(\omega^*, a_2). \tag{20}$$

For the outcome to be ω^*-efficient, the receiver must also be indifferent:

$$u_R(\omega^*, a_1) = u_R(\omega^*, a_2). \tag{21}$$

If this indifference did not hold, then deviating to a pure action would strictly improve the receiver's payoff in the state ω^*. Consider any perturbation of payoffs that preserves the sender's indifference, so the outcome remains an equilibrium. For any such generic perturbation, the receiver's indifference conditions hold only on a subset of payoff vectors with Lebesgue measure zero, and thus represent a non-generic condition. For example, if $A = \{a_1, a_2\}$, both indifference conditions imply that the state ω^* is payoff-irrelevant.

Second, consider the case where the signaling policy is pure. As the outcome is stochastic, this implies that the receiver's response to some message m is mixed. This implies that the receiver has multiple best responses at the posterior belief q_m. Again, to ensure efficiency, assume exactly two actions a_1 and a_2 are played with positive probability and the sender is indifferent between them. Otherwise, deviating to a pure action would strictly improve the sender's payoff without reducing the receiver's. This implies that

$$\mathbb{E}_{q_m}[u_S(\omega, a_1)] = \mathbb{E}_{q_m}[u_S(\omega, a_2)], \qquad \mathbb{E}_{q_m}[u_R(\omega, a_1)] = \mathbb{E}_{q_m}[u_R(\omega, a_2)]. \tag{22}$$

As before, this corresponds to a non-generic condition. So, the cheap talk outcome is inefficient with respect to the prior q_m under all generic perturbations. This implies that there exists a deviation $d_\mu(\omega, a)$ in some state $\omega \in \text{supp}(q_m)$ and action a that violates the efficiency condition of Proposition 2. First, since $\text{supp}(q_m) \subseteq \text{supp}(p)$, this deviation is also feasible under the prior p. Second, condition (ii) in Proposition 2 imposes stronger restrictions under prior p as the set of possible deviations is larger. Hence, generically, the outcome is inefficient with respect to F_p.

□

Intuitively, any stochastic cheap talk outcome relies on knife-edge indifferences for the players. These break under generic payoffs, so only pure outcomes can be efficient. Kamenica and Lin (2025) show that, generically, if the sender's preferred cheap talk outcome is necessarily stochastic, then he values commitment. Hence, the cheap talk outcome must be pure not only for efficiency but also for commitment to have no value.

Next, we consider the case where the sender's payoff is state-independent. Lipnowski and Ravid (2020) characterize the sender's preferred equilibrium using a belief-based approach. Define the receiver's best responses given his belief $p \in \Delta\Omega$ as $A^*(p) := \arg\max_{a\in A} \mathbb{E}_p[u_R(\omega, a)]$. The sender's value function

$$V(p) := \max_{a \in A^*(p)} \mathbb{E}_p[u_S(\omega, a)], \tag{23}$$

represents the sender's expected payoff when the receiver, with belief p, selects the sender's preferred best response. They show that the sender's preferred equilibrium corresponds to the quasiconcave envelope of the value function, evaluated at the prior, which we denote by Quasicav V. We graphically illustrate this equilibrium for Example 1 in Figure 2.

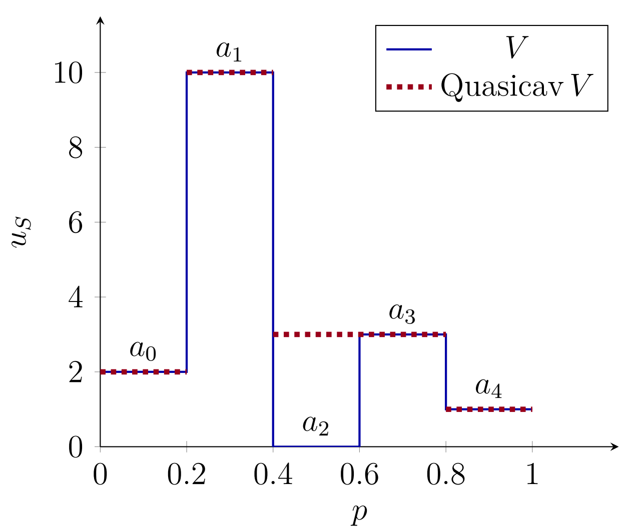

Figure 2: Cheap talk: The value function (blue solid) and its quasiconcave envelope (red dotted). Vertical lines show the jumps at cutoff beliefs.

Let $A^* := \bigcup_{p \in \Delta\Omega} A^*(p)$ denote the set of actions that are a best response for the receiver under some belief, and let $a^* := \arg\max_{a \in A^*} u_S(a)$ denote the sender's most preferred action in A^*. To avoid non-generic cases, we assume that the sender's payoffs are distinct across actions, i.e., $u_S(a) \neq u_S(b)$ whenever $a \neq b$.

Proposition 4. *For a sender with state-independent payoff, a cheap talk outcome is efficient if and only if the sender's preferred action a^* is induced with certainty.*

Proof. ($\Rightarrow$) Following Sobel (2013), we say an equilibrium is *influential* if the receiver does not always take the same action. First, we show that any influential equilibrium is inefficient. Then, we show that any non-influential equilibrium that does not induce the sender's preferred action with certainty is inefficient.

Assume that the equilibrium is influential, that is, $\tau(m) \in \Delta A$ is not constant on the equilibrium path. For this to happen, at least two messages are sent with positive probability, resulting in different actions. First, observe that at least one message must induce a non-degenerate posterior belief $q \in \Delta\Omega$.[7] This is due to the sender's equilibrium condition as he must be indifferent between sending messages that result in the same expected payoff. And as we assume the sender's payoff is state-independent and each action leads to a different payoff, some randomization is necessary for the indifference condition to hold. Such a randomization can only occur at a non-degenerate belief where the receiver has multiple best response actions, that is, $|A^*(q)| > 1$.[8] So, there must be at least two distinct actions a_1 and a_2 that are played with positive probability when the posterior belief q is induced. For example, consider the influential equilibrium for prior $p = 0.5$ in Figure 2, where the posterior beliefs $q = 0.4$ and $q = 0.6$ are induced. Given the non-degenerate posterior belief $q = 0.4$, the sender must randomize between a_1 and a_2 to satisfy the sender's indifference condition. However, the sender strictly prefers one action over the other, for instance, assume $u_S(a_1) > u_S(a_2)$. This implies that the sender's preferred action a_1 is induced with less than probability one at belief $q = 0.4$. Since both actions are receiver's best responses under some non-degenerate belief, there exists a hyperplane passing through the belief q that separates the simplex into two convex regions—one where the receiver prefers a_1 and one where he prefers a_2. In our example, for the pair of actions a_1 and a_2, these convex regions are given by the intervals $[0, 0.4]$ and $[0.4, 1]$ respectively. Using this partition, one can always identify a state where the receiver also prefers the sender's preferred action. Formally, given actions $a_1, a_2 \in A^*(q)$, there exists a state ω^* such that $q(\omega^*) > 0$ and $u_R(\omega^*, a_1) \geq u_R(\omega^*, a_2)$. In our example (see Figure 2), both the sender and the receiver prefer action a_1 over a_2 in state $\omega^* = \omega_0$.

[7] A non-degenerate belief refers to a belief where the probability distribution assigns positive probability to more than one state.

[8] We omit non-generic cases where the receiver has multiple best responses at any degenerate belief.

Hence, the equilibrium is not ex-post efficient in state ω^*. The deviation to play the sender's preferred action a_1 is profitable and does not satisfy condition (i) of Proposition 2.

Now, consider a non-influential (or babbling) equilibrium where the receiver always plays the action $a \neq a^*$.[9] As we assume the prior lies in the interior of the belief simplex, as before, we can identify a state ω^* where the receiver prefers the sender's preferred action a^* over a. For example, given a prior $p \in [0.6, 0.8]$ in Figure 2, the equilibrium is babbling, resulting in action a_3 with certainty. However, both players prefer the action $a^* = a_1$ over a_3 in state $\omega^* = \omega_0$. So, the equilibrium is not ex-post efficient in state ω^*. To summarize, any cheap talk equilibrium that does not induce the sender's preferred action a^* with certainty is inefficient.

($\Leftarrow$) The babbling equilibrium where the sender's preferred action a^* is induced with certainty is an efficient outcome. In this case, the sender gets the highest payoff within his feasible set, ensuring that the outcome lies on the Pareto frontier.

$\square$

Therefore, in cheap talk models with state-independent sender payoffs, we can precisely determine the efficiency of any equilibrium outcome. An outcome is efficient if and only if it corresponds to the babbling equilibrium, in which the receiver plays the sender's preferred action a^*. In this case, the outcome would have been efficient even without communication. Thus, any non-trivial equilibrium in which communication affects the receiver's action necessarily leads to inefficiency.

3.1.2 Bayesian persuasion

In Bayesian persuasion, the sender commits to his signaling policy prior to observing the state of the world. This contrasts with cheap talk, where the sender cannot commit and chooses a message after observing the state. Our goal is to analyze the efficiency of the equilibrium outcome of Bayesian persuasion.

The sender's objective is to persuade the receiver to take actions that maximize his ex-ante expected payoff. As is standard in the literature, we assume that the ties are broken in favor of the sender. It is without loss of generality to restrict attention to direct signaling policies. Given prior $p \in \Delta\Omega$, let μ_p^* denote the equilibrium outcome of Bayesian persuasion. This outcome and the corresponding direct policy are generically unique.

Definition 6. *The* ***Bayesian persuasion (BP) outcome*** $\mu_p^* : \Omega \to \Delta A$ *solves the*

[9] If the receiver chooses a mixed action, select a pure action from the support that differs from the sender's preferred action, that is, pick action $a \in \bigcup_m \operatorname{supp}(\tau(m))$ such that $a \neq a^*$.

following linear program:

$$\max_{\mu:\Omega\to\Delta A} \sum_{\omega\in\Omega} p(\omega) \sum_{a\in A} \mu(a \mid \omega) u_S(\omega, a) \tag{24}$$

subject to

$$\sum_{\omega\in\Omega} p(\omega)\mu(a \mid \omega)\big[u_R(\omega, a) - u_R(\omega, b)\big] \geq 0 \quad \forall a, b \in A. \tag{25}$$

Equation (25) corresponds to the receiver's *obedience* condition: given a recommended action a, the receiver prefers following it to deviating to any other action b.

In Kamenica and Gentzkow (2011), the BP outcome is characterized using the concavification approach (Aumann et al., 1995). Denote by $\mathrm{Cav}\ V : \Delta\Omega \to \mathbb{R}$ the concave envelope of the value function V. The sender's expected payoff in the BP outcome is given by the evaluation of the concave envelope at the prior: $\mathbb{E}_{\mu_p^*}[u_S(\omega, a)] = \mathrm{Cav}\ V(p)$.

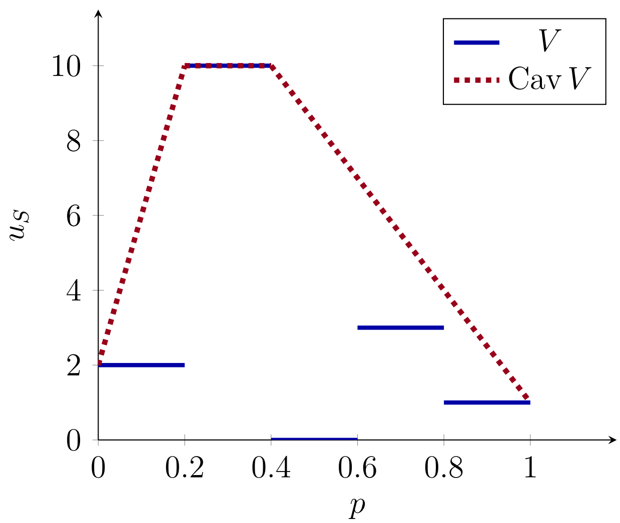

Figure 3: Bayesian persuasion: The value function (blue solid) and its concave envelope (red dotted).

The value function and its concave envelope for Example 1 are depicted in Figure 3. The outcomes in Example 1 correspond precisely to the BP outcomes for the respective priors: (a) $p = 0.10$, (b) $p = 0.30$, and (c) $p = 0.70$. As seen in Figure 1, the BP outcome is efficient for case (b), not for cases (a) or (c). Moreover, since the support of the outcome remains constant within the convex regions defined by the concave envelope, the BP outcome μ_p^* is efficient for $p \in [0.2, 0.4]$ and inefficient for $p \in (0, 0.2) \cup (0.4, 1)$.

Buyer-Seller Interactions: In this section, we examine a natural class of buyer-seller interactions, where the necessary condition for efficiency is not satisfied by the BP outcome for a wide range of parameters. First, by fixing the receiver's preferences we find a large range of priors for which the BP outcome is inefficient. On the other hand, fixing a prior we show that there is a range of preferences for which the BP outcome is inefficient.

For motivation, consider the following situation: a seller (S) provides information to a buyer (R) about which product to purchase, if any, from a set of n products. The buyer chooses from the action set $A = \{0, 1, ..., n\}$, where action i for $i = 1, ..., n$ corresponds to buying product i and action 0 corresponds to not buying any product. The state space is given by $\Omega = \{\omega_0, \omega_1, ..., \omega_n\}$, where ω_i for $i = 1, ..., n$ denotes the state when it is optimal for the buyer to purchase product i and ω_0 denotes the state when it is optimal for the buyer to not purchase any product. The seller's payoff is state-independent. He gets $u_S(i) > 0$ if product i is bought and gets zero if no product is bought, that is, $u_S(0) = 0$. The seller and buyer share a common prior $p \in \text{int}(\Delta\Omega)$, where the buyer's optimal action under the prior is to not purchase anything.

Let $C_i \subset \Delta\Omega$ denote the convex subset of the belief space where the receiver's optimal action is i. The collection of these subsets for all actions $\{C_i\}_{i\in A}$ forms a partition $\mathcal{P} = \{C_0, C_1, \ldots, C_n\}$ of the belief space $\Delta\Omega$ (see Figure 4).

$$C_i := \{p \in \Delta\Omega : \mathbb{E}_p[u_R(\omega, i)] \geq \mathbb{E}_p[u_R(\omega, j)] \quad \forall j \in A\}. \tag{26}$$

We consider a partition $\mathcal{P} = \{C_0, C_1, \ldots, C_n\}$ of the $n + 1$ convex sets (induced by receiver's preferences) that satisfy the following properties:

1. The receiver's optimal action in state ω_i is i, i.e., $\omega_i \in C_i \quad \forall i = 0, \ldots, n$. To avoid non-generic situations, we further assume that there is an open neighbourhood N_{ω_i} such that $\omega_i \in N_{\omega_i} \subset C_i$.

2. For any pair of distinct products i and j ($j \neq i$ and $i, j \neq 0$), $C_i \cap C_j = \emptyset$.

Condition 1 ensures that it is optimal to take action i when the receiver is sufficiently confident that the state is ω_i. Condition 2 ensures that when the receiver is unsure whether the product is i or j, he prefers not to buy either product (i.e., take action a_0).

To characterize the BP outcome μ_p^*, we follow Lipnowski and Mathevet (2017) and restrict the feasible posteriors to the finite set of outer points $Out(\mathcal{P})$ of the partition $\mathcal{P}$.

$$Out(\mathcal{P}) :=\{p \in \Delta\Omega : p \in ext(C_i) \text{ whenever } p \in C_i \in \mathcal{P}\}, \tag{27}$$

$$=(\bigcup_{i=0}^{n}\omega_i) \cup (\bigcup_{i=1}^{n}\bigcup_{j\neq i} o_{ij}), \tag{28}$$

where o_{ij} is the (unique) extreme point of the convex set C_i that lies on the line segment joining vertices ω_i and ω_j.[10] Lipnowski and Mathevet (2017) shows that the equilibrium outcome can always be induced using this set of posterior beliefs $Out(\mathcal{P})$. They further show that we only need to consider an affinely independent collection of these beliefs. In our setting, this implies that the equilibrium signaling policy does not need to induce more than $n+1$ posterior beliefs.

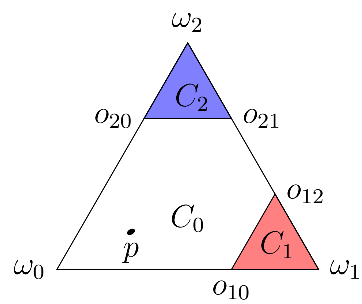

Figure 4: Receiver's belief space for $n = 2$.

Let $n = 2$ and the prior p be as shown in Figure 4. Any feasible outcome, where the posterior belief belongs to the set of outer points, induces all three actions. Under the BP outcome, the seller fully reveals state ω_0 leading the buyer to take action a_0. However, the seller only partially reveals the other states, inducing both actions a_1 and a_2 with positive probability in states ω_1 and ω_2, respectively. The posteriors in the BP outcome cannot place positive weight on nodes ω_1 and ω_2, since another feasible outcome that the sender strictly prefers is available. This deviation increases the probability of the seller's preferred actions a_1 and a_2 and reduce that of the least preferred action a_0. This is achieved by inducing the beliefs $o_{10}, o_{12}, o_{20}, o_{21}$ instead of ω_1 and ω_2. As a result, the BP outcome is inefficient, as it is necessarily mixed. Either all three actions a_0, a_1, and a_2 occur in state ω_0, or actions a_1 and a_2 are both played in states ω_1 and ω_2, violating the necessary condition of Theorem 1.

We generalize this idea for n products and show that it is always possible to find a set $R_* \subset C_0$, where $\dim(R_*) = \dim(\Delta\Omega) = n$, such that the BP outcome μ_p^* is inefficient for any prior $p \in R_*$. The crux of the proof relies on finding a region where the BP outcome induces

[10]As we break ties in favor of the seller, the buyer takes action a_i (where $u_S(i) > u_S(j)$) if the belief o_{ij} is induced.

at least three actions which cannot have support on any vertex ω_i for $i \neq 0$. We show this violates the necessary condition in Proposition 1.

Proposition 5. *For $n \geq 2$, there exists a set $R_* \subseteq C_0$ such that the BP outcome μ_p^* is inefficient for all $p \in int(R_*)$.*

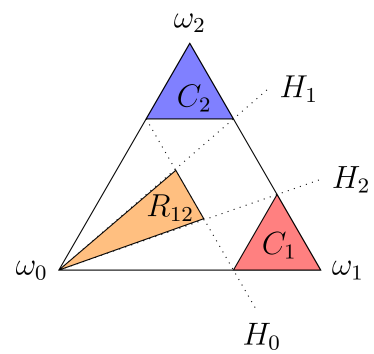

Figure 5: The set R_{12} (orange region) for $n = 2$.

Proof. Let H_i for $i \neq 0$ be the hyperplane defined by the set of points $\{\omega_0\} \bigcup_{j\neq i,0}\{o_{ji}\}$ (see Figure 5). The hyperplane H_i separates the convex set C_i from C_j for all $j \neq i, 0$. Denote by R_i the region in the simplex given by the half-space of H_i that includes C_i. For any prior $p \in R_i$, it is necessary that any feasible outcome induces action i. Similarly, let H_0 be the hyperplane defined by $\{o_{10}, ..., o_{n0}\}$ and R_0 denote the convex region in the simplex that includes the node ω_0 and is separated by the half-space of H_0. For any prior $p \in R_0$, it is necessary to play action 0 under any feasible outcome.

For any $i, j \neq 0$ let $R_{ij} = R_i \cap R_j \cap R_0$ (see Figure 5). The convex set R_{ij} is non-empty as $\omega_0 \in R_{ij}$. In fact, we show that $\dim(R_{ij}) = \dim(\Delta\Omega) = n$ for all $i, j \neq 0$. Given a system of inequalities $Ax \leq b$, an inequality $a_i^T x \leq b_i$ in $Ax \leq b$ is an *implicit equality* if $a_i^T \bar{x} = b_i \quad \forall \bar{x} \in \{x : Ax \leq b\}$. A polyhedron $R \subseteq \mathbb{R}^l$ has full dimension ($\dim(R) = l$) if and only if it has no implicit equality (see Conforti et al., 2014). The polyhedron R_{ij} is defined by the system of inequalities of the hyperspaces R_0, R_i and R_j. If the polyhedron R_{ij} has an implicit equality, then all points $p \in R_{ij}$ lie on the hyperplane H_0, H_i or H_j. But this happens only if $o_{ij} = o_{ji}$ or $o_{k0} = \omega_0$ where $k = i, j$. But as we assume (1) there is an open neighbourhood $N_{\omega_0} \subset C_0$ and (2) $C_i \cap C_j \neq \emptyset$, we can conclude there is no implicit equality for the polyhedron R_{ij}. Thus, we have $\dim(R_{ij}) = \dim(\Delta\Omega) = n$.

Given prior $p \in R_{ij}$, any feasible outcome induces the actions i, j and 0. We claim that posteriors of the BP outcome cannot include the vertices ω_i or ω_j. We prove by contradiction, assume the feasible outcome μ_1 is optimal and its induced posteriors include the vertex ω_i. As $p \in R_0$, it needs to induce action 0 and its support includes the node ω_0. Now, as $o_{i0} \in (\omega_0, \omega_i)$ and is separated from p by the hyperplane H_0, there exists a feasible outcome μ_2, where the belief o_{io} is induced instead of ω_i. This outcome leads to a higher probability of action i and conversely a lower probability of action 0. Let λ_i denote the weight of outcome μ_i on its posteriors. We have $\lambda_2(o_{io}) = \frac{\lambda_1(\omega_i)}{o_{io}(\omega_i)} > \lambda_1(\omega_i)$ and $\lambda_2(\omega_0) = \lambda_1(\omega_0) - \frac{\lambda_1(\omega_i)o_{i0}(\omega_0)}{o_{i0}(\omega_i)} < \lambda_1(\omega_0)$. The weight on all other actions $j \neq i$ remains the same. Thus, μ_2 is a profitable deviation and the posteriors of the BP outcome cannot include ω_i or ω_j.

Using the above result, for any prior $p \in R_{ij}$, the BP outcome μ_p^* has either (a) three actions i, j and 0 played in state ω_0 (eg: $\bigcup_{m \in M} q_m = \{\omega_0, o_{10}, o_{20}\}$ in Figure 4) or (b) mixed outcomes in states ω_i and ω_j (eg: $\bigcup_{m \in M} q_m = \{\omega_0, o_{12}, o_{21}\}$ in Figure 4). This violates the necessary condition for efficiency in Theorem 1. The set $R_* = \bigcup_{j \neq 0} \bigcup_{i \neq j,0} R_i \cap R_j \cap R_0$ combines the regions R_{ij} for all pairs of distinct products i and j.

□

In the previous proposition, we fixed the underlying preferences of the players and looked at priors for which the BP outcome is inefficient. In the next proposition, we fix a prior and look at partitions induced by the receiver's preferences for which the BP outcome is inefficient.

Consider a receiver who only buys a product when he is sufficiently confident about the state, i.e., he only buys product i if his belief on state ω_i is above a certain threshold T (where $T > 0.5$).[11] The threshold can be interpreted as the receiver's risk attitude (for buying a product). A higher T implies a receiver who is more risk averse. The convex subset C_i, where the receiver's optimal action is i, is given by:

$$C_i = \{p \in \Delta\Omega \mid p(\omega_i) \geq T\} \quad \forall i = 1, \dots, n, \tag{29}$$

$$C_0 = \Delta\Omega \setminus \bigcup_{i=1}^{n} C_i. \tag{30}$$

Let $\mathcal{P}_T = \{C_0, ..., C_n\}$ denote the partition where the receiver's preference is given by threshold T. We show that for any prior p, there exists a bound T_p such that for any threshold $T > T_p$, the BP outcome is inefficient.

Proposition 6. *For any prior $p \in int(\Delta\Omega)$, there exists a threshold $T_p < 1$ such that the BP outcome μ_p^* is inefficient with respect to the partition $\mathcal{P}_T$ for all $T > T_p$.*

Proof. First, we show for any $p \in \text{int}(\Delta\Omega)$ and action i, there exists a threshold T_p^i such that whenever $T > T_p^i$, any feasible outcome induces action i. Using this characterization, we then show that there exist receiver preferences under which the BP outcome μ_p^* is inefficient for the partition $\mathcal{P}_T$.

[11] A similar class of preferences is considered in Aybas and Turkel (2019).

Let q denote the point of intersection between the line joining the points ω_0 and p and the face $F_0 = \{p \in \Delta\Omega : p(\omega_0) = 0\}$. Recall, the hyperplane H_i defined by the points $\{\omega_0\} \bigcup_{j \neq i,0} \{o_{ji}\}$. It separates C_i and the convex sets $\{C_j\}_{j \neq i,0}$. The point p lies in the region R_i if $T \geq T_p^i = 1 - q(\omega_i)$ (see Figure 6). This follows as $p \in R_i$ if and only if it's projection $q \in R_i$. And q belongs to R_i (for $i \neq 0$) if and only if $q(\omega_i) \geq 1 - T$. Similarly, the hyperplane H_0 separates the vertex ω_0 and the convex sets $\{C_j\}_{j \neq 0}$ and we have $p \in R_0$ if $T \geq T_p^0 = 1 - p(\omega_0)$.

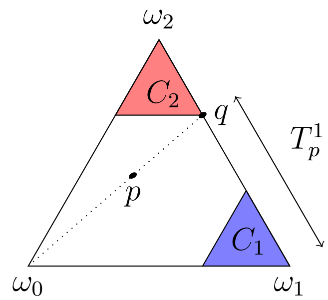

Figure 6: Threshold beliefs for $n = 2$.

Let $i^* = \underset{i \neq 0}{\operatorname{argmax}}\ q(\omega_i)$ and let $j^* = \underset{i \neq 0, i^*}{\operatorname{argmax}}\ q(\omega_i)$. The vertices ω_{i^*} and ω_{j^*} represent the states that are closest to the projection $q \in \Delta\Omega$. We have $p \in \operatorname{int}(R_{i^*} \cap R_{j^*} \cap R_0)$ if

$$T > T_p = \max\{1 - p(\omega_0), 1 - q(\omega_{i^*}), 1 - q(\omega_{j^*})\}. \tag{31}$$

Since the common prior $p \in \operatorname{int}(\Delta\Omega)$, we have $p(\omega_0) > 0$ and $q(\omega_i) < 1$ for $i \neq 0$. Hence each term in Equation (31) is strictly less than 1, and therefore $T_p < 1$.

Therefore, whenever $T > T_p$, the Bayesian persuasion outcome μ_p^* lies in R_* and is inefficient with respect to the partition $\mathcal{P}_T$.

□

3.2 Peer Selection Mechanisms

In this section, we analyze the mechanism design problem without transfers studied in Niemeyer and Preusser (2024). A single indivisible good needs to be allocated. There are k players: one principal and $k - 1$ agents, where $k \geq 3$. The state is denoted by $\omega = (\omega_1, \ldots, \omega_{k-1}) \in \Omega$, where $\Omega = \prod_{i=1}^{k-1} \Omega_i$. Agent i's private type is $\omega_i \in \Omega_i$, where $|\Omega_i| \geq 2$. If agent i receives the good, his payoff is 1 and 0 otherwise. The principal's payoff from allocating to agent i is a type-dependent value $u_i \in [-1, 1]$, while keeping the good his payoff is 0. Let p denote the common prior over the set of states (type profiles). Let $A = \{0, 1, \ldots, k - 1\}$ denote the set of pure actions, where $a = 0$ means the principal keeps the good and $a = i$ means agent i receives it. The payoff of the players depends only on the state and the action.

A (direct) mechanism is a mapping $\mu : \Omega \to \Delta A$, where $\mu(a \mid \omega)$ is the probability that action a is taken when the state is ω. Dominant-strategy incentive compatibility (DIC) requires that truthful reporting maximizes each agent's allocation probability:

$$\mu(i \mid \omega_i, \omega_{-i}) \geq \mu(i \mid \omega_i', \omega_{-i}), \quad \forall \omega_i, \omega_i' \in \Omega_i,\ \forall \omega_{-i} \in \Omega_{-i}, \forall i \in \{1, \ldots, k-1\}. \tag{32}$$

The principal's goal is to find the *optimal mechanism*: the DIC mechanism that maximizes his expected payoff. Because the objective is linear and the set of DIC mechanisms is convex, any optimal mechanism is a convex combination of *extreme* DIC mechanisms. In particular, they show that when the state space is sufficiently rich, essentially all extreme mechanisms are stochastic. Consequently, randomization is an inherent structural feature of optimal mechanisms.

Ranking-based mechanism Because optimal mechanisms lack closed-form descriptions, Niemeyer and Preusser (2024) introduce *ranking-based mechanisms*, which are simple yet approximately optimal. These mechanisms work in two steps. First, fix a threshold $t \in (0, 1]$ and let $r_i(\omega)$ denote agent i's rank at state ω, normalized on the scale $\{\frac{1}{k-1}, \frac{2}{k-1}, \ldots, 1\}$ according to *peer value* $u_i(\omega_{-i}) := \mathbb{E}[u_i \mid \omega_{-i}]$. The principal randomly selects one of these eligible agents ranked among the top $t(k-1)$ agents. Second, the principal allocates the good to the selected agent if and only if his *robust rank* $r_i^*(\omega_{-i}) = \max_{\omega_i \in \Omega_i} r_i(\omega_i, \omega_{-i})$ also lies within the top $t(k-1)$ and his peer value is non-negative. If the selected agent fails this eligibility test, the good is kept by the principal. Thus, each eligible agent is assigned the good with a probability of at most $1/(t(k-1))$, and any leftover probability mass is kept by the principal.

A crucial factor for the performance of ranking-based mechanisms is the impact that any individual agent's report has on his own rank, which they refer to as the *informational size*. For each state ω, the informational size is defined as

$$\delta(\omega) := \max_{i \in \{1, \ldots, k-1\}} \max_{\omega_i' \in \Omega_i} \left| r_i(\omega_i, \omega_{-i}) - r_i(\omega_i', \omega_{-i}) \right|. \tag{33}$$

When informational size is large, many agents can manipulate their position above the threshold, and the mechanism may erroneously withhold the good. When it is small, no single agent can substantially affect rankings. This assumption is natural in large environments with many agents, where each individual's type has only limited influence.

We impose some mild conditions, similar to Niemeyer and Preusser (2024). First, for every state, there exists at least one agent with non-negative peer value, and the threshold is chosen such that $t > \frac{1}{k-1} + \delta(\omega)$. Together, these assumptions guarantee that there is at least one eligible agent to whom the principal will allocate the good if selected. Second, to avoid trivial situations, we assume $t(k-1) \geq 2$, so that there are at least two eligible agents in every state. Ranking-based mechanisms are approximately optimal when the informational size is small and there are a large number of agents. We show that, under similar assumptions, these mechanisms are generically inefficient.

Proposition 7. *Suppose that for every state ω, the threshold t satisfies $t \geq \max\left\{\frac{1}{k-1} + \delta(\omega), \frac{2}{k-1}\right\}$, and that there exists at least one agent i with non-negative peer value $u_i(\omega_{-i}) \geq 0$. Then, generically, the ranking-based mechanism is inefficient.*

Proof. Under our assumptions, in the first step the principal randomly selects among at least two agents, i.e., $t(k-1) \geq 2$. At least one of these agents, if chosen, is allocated the good. Hence, with positive probability, either two distinct agents receive the good or one agent and the principal do. Thus, for all $\omega \in \Omega$, we have $|\mu(\omega)| \geq 2$. Moreover, since each agent has at least two types, $|\Omega| = \prod_{i=1}^{k-1} |\Omega_i| \geq 2^{k-1}$. Therefore, $\sum_{\omega \in \Omega} |\mu(\omega)| \geq 2|\Omega| > k + |\Omega|$, where the last inequality uses $2^{k-1} > k$ for $k \geq 3$. This contradicts the bound in Theorem 1, so the outcome induced by the ranking-based mechanism is inefficient. □

4 Conclusion

This paper analyzed Pareto efficiency in games with incomplete information. We identified necessary conditions based on the number of actions taken across states—independent of the prior and the extent of randomization. We found that any stochastic cheap talk outcome is generically inefficient, and when the sender's preferences are state-independent, it is efficient if and only if the sender's preferred action is chosen with certainty. In Bayesian persuasion, equilibrium outcomes are generically inefficient for a broad set of priors and preferences. Our results highlight that preference misalignment prevents efficiency in direct communication between the sender and receiver. Beyond two-player settings, our results extend to mechanism design, where unavoidable randomization generically leads to inefficiency.

Several directions for future research remain open. A key challenge is to identify sufficient conditions under which persuasion outcomes can be efficient. Moreover, alternative communication models—such as mediation or delegation—warrant further study. A central question is whether some communication protocol can guarantee efficiency.

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