

# Efficiency in Games with Incomplete Information

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AMES 2026-CSW

January 23, 2026

# Motivation

- Pareto efficiency is a natural benchmark for social welfare: no individual can be made strictly better off without making another worse off.
- Under incomplete information, players possess private information that could improve collective welfare.
- However, individual incentives may lead them to distort what they know.
- **When is a feasible outcome Pareto efficient? When can strategic behavior lead to efficiency?**

## Summary of Results

- Excessive randomization over action profiles leads to inefficiency.
- **Necessary condition for efficiency (generic):**  
 $\# \text{ action profiles across states} < \# \text{ players} + \# \text{ states}$ .  
Two players: only pure or quasi-pure outcomes can be efficient.

## Economics Applications

1. Cheap talk
2. Bayesian persuasion
3. Allocation problem without transfers

## Finite game with incomplete information

- **Players:**  $i \in \{1, \dots, k\}$ .
- **States:**  $\omega \in \Omega$  drawn from prior  $p \in \text{int}(\Delta\Omega)$ .
- **Actions:**  $a = (a_1, \dots, a_k) \in A = \prod_{i=1}^k A_i$ .
- **Payoffs:**  $u_i : \Omega \times A \rightarrow \mathbb{R}$ .
- **Types:**  $t = (t_1, \dots, t_k) \in T = \prod_{i=1}^k T_i$ , with  $\pi : \Omega \rightarrow \Delta T$ .

## Outcomes and feasible payoffs

- **Outcome:**  $\mu : \Omega \rightarrow \Delta A$ .
- **Induced ex-ante payoff:**

$$u(\mu) = \sum_{\omega \in \Omega} p(\omega) \sum_{a \in A} \mu(a \mid \omega) u(\omega, a) \in \mathbb{R}^k.$$

- **Feasible payoffs (given  $p$ ):**

$$F_p = \{u(\mu) \in \mathbb{R}^k : \mu : \Omega \rightarrow \Delta A\}.$$

The set  $F_p$  is a convex polytope; extreme points = pure outcomes.

## Efficiency

- An outcome  $\mu$  is **efficient** if there is no outcome  $\eta$  with  $u(\eta) \geq u(\mu)$  and at least one strict inequality.
- $\mu$  is efficient  $\iff$  it maximizes a **strictly positive weighted sum** of players' ex-ante payoff:

$$\exists n \in \mathbb{R}_{++}^k \text{ s.t. } u(\mu) \in \arg \max_{x \in F_p} n^\top x.$$

## Necessary Condition for Efficiency

Let  $|\mu(\omega)|$  denote number of action profiles taken in state  $\omega$ .

Generically, an outcome  $\mu$  is efficient only if  $\sum_{\omega} |\mu(\omega)| < k + |\Omega|$ .

**Two players:** generically, efficient outcomes are pure or quasi-pure (binary randomization in exactly one state).

This condition does not depend on the prior, which action profiles are used, or the weight of randomization.

## Ex-ante and ex-post efficiency: a geometric link

- **Minkowski sum decomposition:**

$$F_p = \sum_{\omega \in \Omega} p(\omega) F_\omega, \text{ where } F_\omega = \text{Co}\{u(\omega, a) : a \in A\}.$$

- **Ex-ante efficiency:**  $u(\mu)$  lies on the Pareto frontier of  $F_p$ .
- **Ex-post efficiency:**  $u(\mu | \omega)$  lies on the Pareto frontier of  $F_\omega$ .
- **Key geometric result:**

$$\mu \text{ is efficient} \iff \exists n \in \mathbb{R}_{++}^k \text{ s.t. } u(\mu | \omega) \in \arg \max_{x \in F_\omega} n^\top x \quad \forall \omega \in \Omega$$

## Intuition for the result

- Efficiency requires a **common** positive weight vector  $n \in \mathbb{R}_{++}^k$  to support **all** states.
- In state  $\omega$ , ex-post efficiency means that all actions played maximize the same linear objective  $n^\top u(\omega, \cdot)$ .
- Generically, each additional action profile imposes an additional independent linear constraint on the admissible weight vectors.
- Summing constraints across states, excessive randomization generically rules out any strictly positive weight vector.

## Example: Sender–Receiver

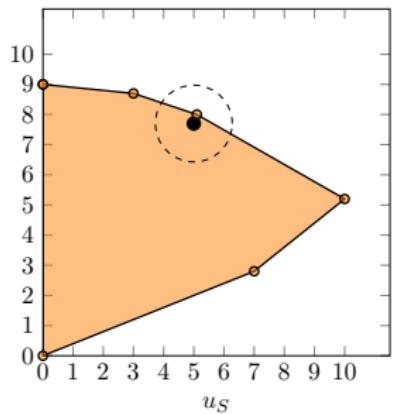
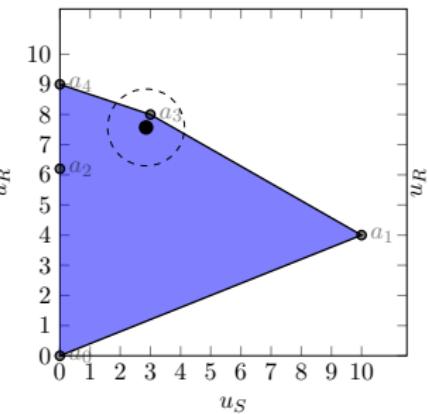
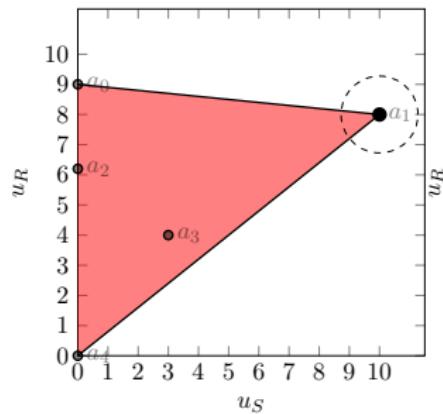
Let  $\Omega = \{\omega_0, \omega_1\}$  and  $A = \{a_0, a_1, a_2, a_3, a_4\}$ . Sender–receiver payoffs  $(u_S, u_R)$ :

	$a_0$	$a_1$	$a_2$	$a_3$	$a_4$
$\omega_0$	(0, 9)	(10, 8)	(0, 6.2)	(3, 4)	(0, 0)
$\omega_1$	(0, 0)	(10, 4)	(0, 6.2)	(3, 8)	(0, 9)

In this environment, we illustrate the efficiency properties of different outcomes.

## Example 1: Ex-post inefficient

1. Outcome where action  $a_1$  is taken in  $\omega_0$ , and actions  $a_1$  and  $a_4$  are taken in  $\omega_1$



$F_{\omega_0}$

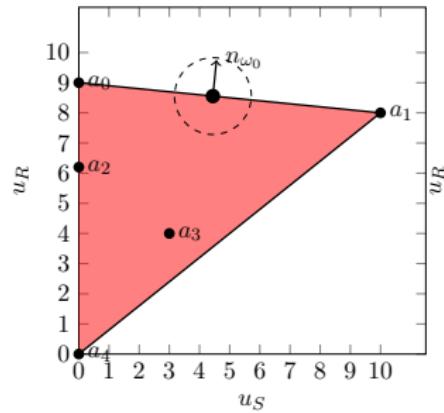
$F_{\omega_1}$

$F_p$

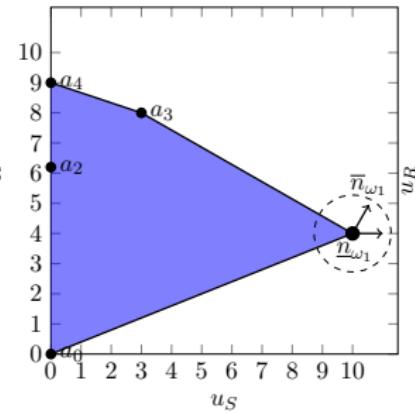
Ex-post inefficient in state  $\omega_1 \Rightarrow$  inefficient

## Example 2: Ex-post efficient but ex-ante inefficient

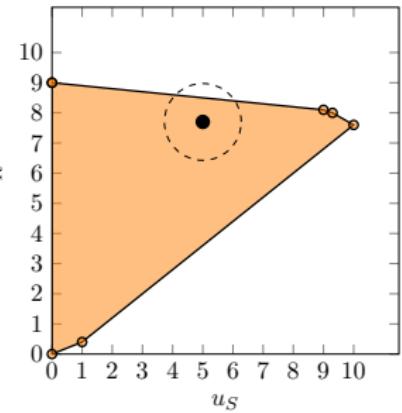
2. Outcome where actions  $a_0$  and  $a_1$  are taken in  $\omega_0$  and action  $a_1$  is taken in  $\omega_1$



$F_{\omega_0}$



$F_{\omega_1}$

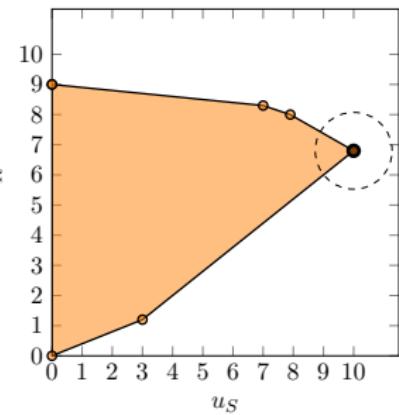
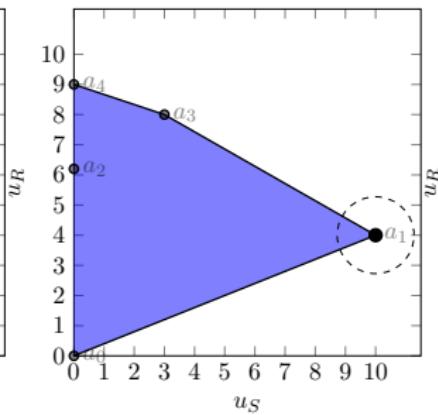
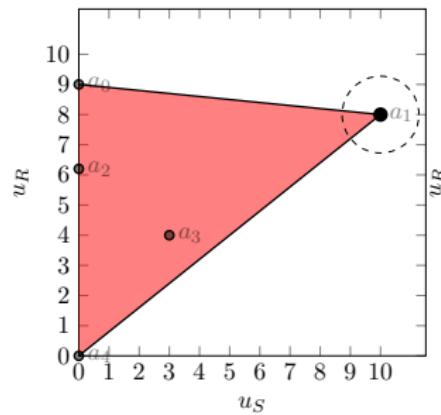


$F_p$

Ex-post efficient in both states but no common positive weight vector ( $\underline{n}_{\omega_0} \notin \text{cone}\{\underline{n}_{\omega_1}, \bar{n}_{\omega_1}\}$ )  $\Rightarrow$  inefficient

## Example 3: Ex-ante efficient

3. Outcome where only action  $a_1$  is taken in both states



$F_{\omega_0}$

$F_{\omega_1}$

$F_p$

Outcome is ex-post efficient in both states and a common positive weight vector exists  $\Rightarrow$  efficient

# Cheap Talk

## Cheap Talk: Model

- State  $\omega \in \Omega$  drawn according to prior  $p \in \text{int}(\Delta\Omega)$ .
- Sender observes  $\omega$  and chooses a message  $m \in M$  to send.
- Upon seeing  $m$ , receiver chooses an action  $a \in A$ .
- Results in payoffs  $u_S(\omega, a)$  and  $u_R(\omega, a)$ .

## When is a cheap talk outcome efficient?

Generically, a cheap talk outcome is efficient only if it is pure.

- Suppose a cheap talk equilibrium induces a **stochastic outcome**.
- Generically, efficiency implies that in some state  $\omega^*$  no more than two actions  $a_1, a_2$  are played.

**Sender's equilibrium:**  $u_S(\omega^*, a_1) = u_S(\omega^*, a_2)$

**Ex-post efficiency:**  $u_R(\omega^*, a_1) = u_R(\omega^*, a_2)$

- Non-generic condition! Any small perturbation breaks this knife-edge indifferences.

## Cheap talk with state-independent sender payoff

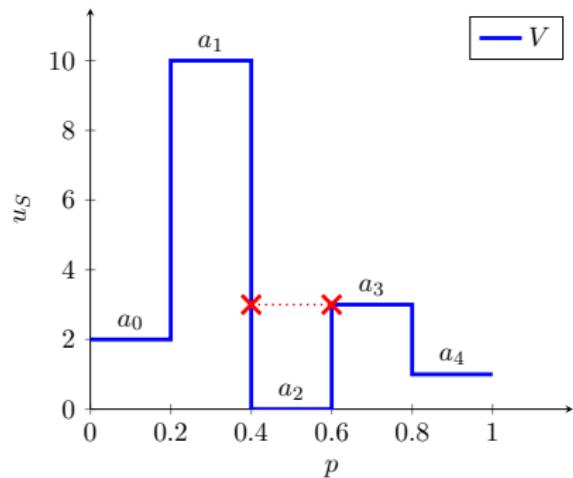
- Let  $a^*$  denote the sender's most preferred action among receiver's best responses.

Suppose  $u_S(a)$ . A cheap talk outcome is efficient  $\iff a^*$  is induced with certainty.

- Any non-trivial cheap talk outcome, where communication affects the receiver's action, is inefficient.

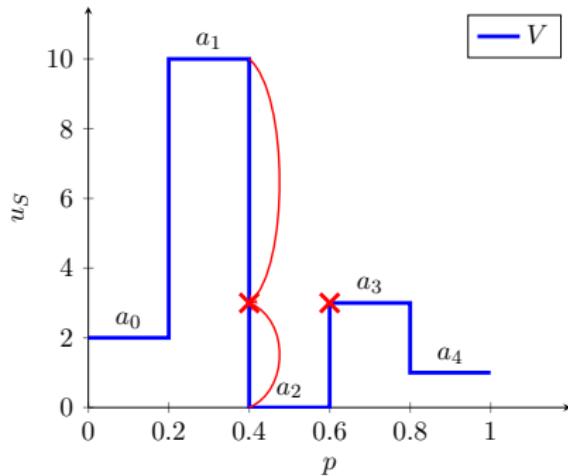
## $u_S(a)$ : Influential equilibrium

(i) Suppose  $p = 0.5$  and posteriors  $q_1 = 0.4$  and  $q_2 = 0.6$  are induced.



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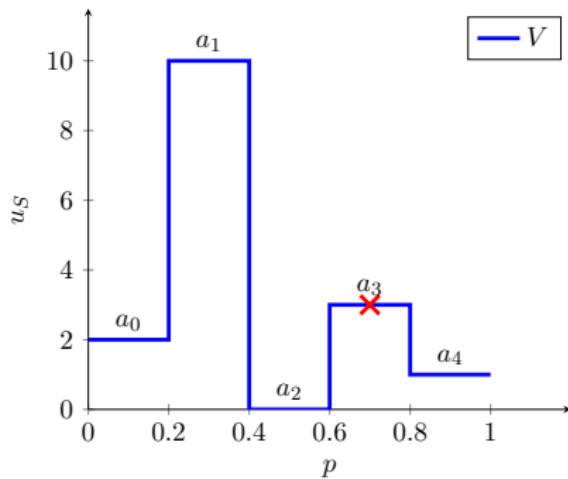


Given posterior  $q_1 = 0.4$ , the receiver must randomize between  $a_1$  and  $a_2$  to satisfy sender's indifference condition.

Both players strictly prefer  $a_1$  over  $a_2$  in state  $\omega_0 \Rightarrow$  inefficient.

## $u_S(a)$ : Babbling equilibrium

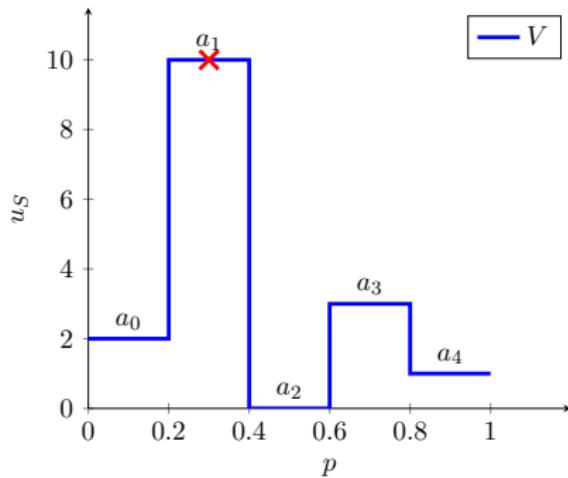
(ii) Suppose  $p = 0.7$  where the action  $a_3 \neq a^*$  is played with certainty.



Again, both players strictly prefer  $a_1$  over  $a_3$  in state  $\omega_0 \Rightarrow$  inefficient

## $u_S(a)$ : Babbling equilibrium

(iii) Suppose  $p = 0.3$  where the action  $a_1 = a^*$  is played with certainty.



This babbling equilibrium is the sender's most preferred outcome  
 $\Rightarrow$  efficient

## Relation to previous literature

- **Pareto efficiency:** Rudov, Sandomirskiy, and Yariv, 2025, Arieli and Babichenko, 2012, Pradelski and Young, 2012, Marden, Young, and Pao, 2014, Jindani, 2022,...
- **Strategic communication:** Ichihashi, 2019, Doval and Smolin, 2024, Crawford and Sobel, 1982, Kamenica and Gentzkow, 2011, ...

## Summary

- We analyzed Pareto efficiency in finite games with incomplete information.
- Generically, excessive randomization across action profiles leads to inefficiency.
- Necessary condition (generic):

$$\# \text{ action profiles across states} < \# \text{ players} + \# \text{ states}.$$

- Incentive constraints often prevent efficiency in natural strategic environments.

**Thank you! Any questions?**